Qiskit-symb: a Qiskit Ecosystem package for symbolic Quantum Computation

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Taipei, Taiwan

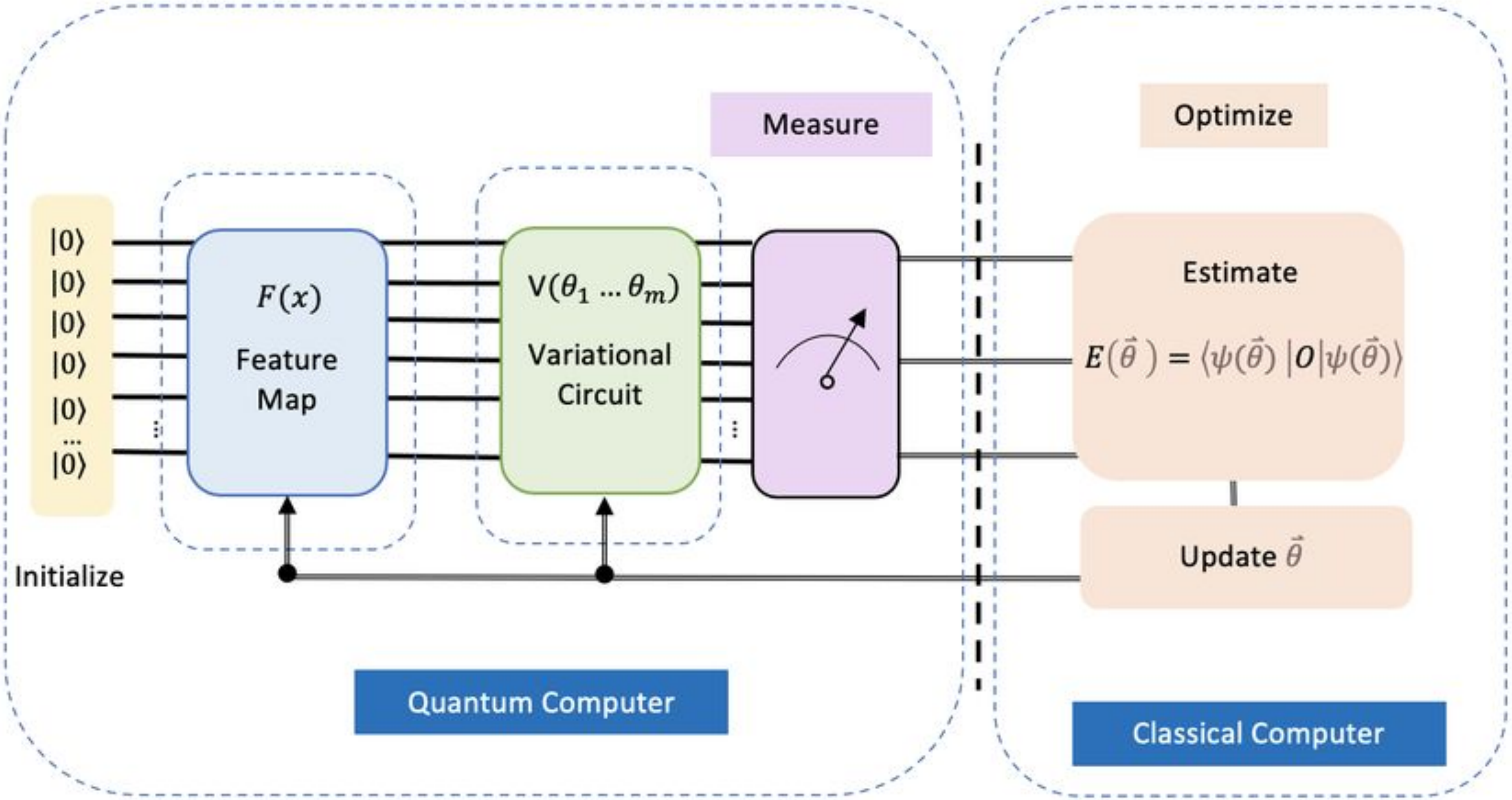
Outline

- Overview of state-of-the-art frameworks for Quantum Machine Learning (QML):
    - Quantum Neural Network
    - Quantum Kernel methods

- Introduction to the **Qiskit Ecosystem** and Qiskit SDK

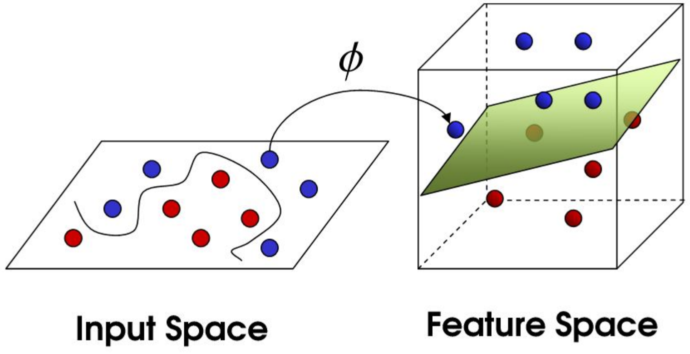
- **Qiskit-symb** for symbolic Quantum Computation in Qiskit
- **Qiskit-symb** for Parameterized Quantum Circuits (PQCs) simulation in QML

Quantum Neural Network



Kernel methods

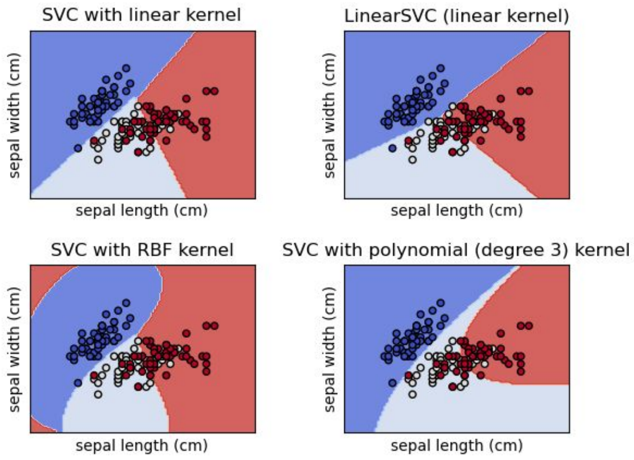
Consider a supervised classification task with complex decision boundaries in the **input space**. A function $\boldsymbol{\phi}$ can be used to map each data point in a higher-dimensional **feature space** with simple decision boundaries.



$$K(x_i, x_j) = \phi^T(x_i) \cdot \phi(x_j)$$

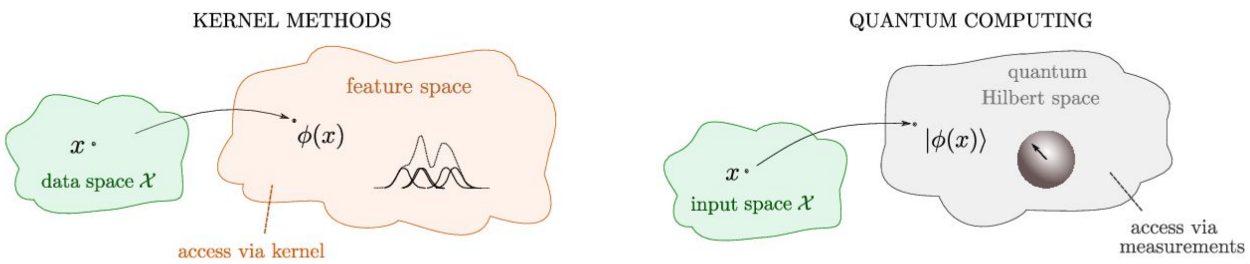
Kernel trick: it's possible to compute the kernel without having to calculate or even know anything about $\boldsymbol{\phi}$

Supervised classification using **SVM model**



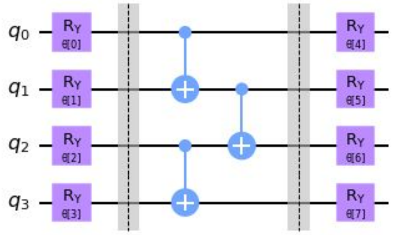
⚠ Computational complexity $\mathrm{O}(\mathrm{N}^2)$, with $\mathrm{N} =$ number of data points

Quantum Kernel



$$|\phi(x)\rangle = U_{\phi(x)}|0^{\otimes n}\rangle$$

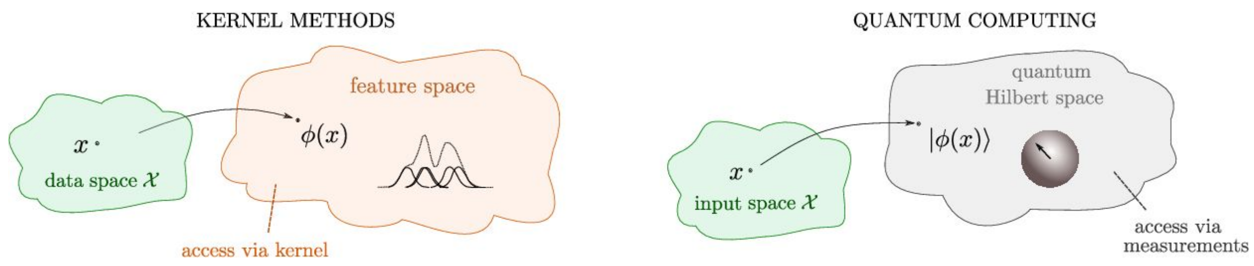
Quantum **feature map** as a PQC



$$K(x_i, x_j) = |\langle \phi^\dagger(x_i)|\phi(x_j)\rangle|^2$$
$$= |\langle 0^{\otimes n}|U^\dagger_{\phi(x_i)} U_{\phi(x_j)}|0^{\otimes n}\rangle|^2$$

The kernel function value is computed as the probability of measuring the state $|0^{\otimes n}\rangle$ at the end of the circuit

Quantum Kernel



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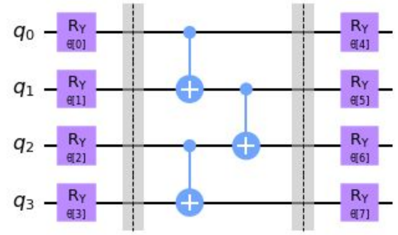

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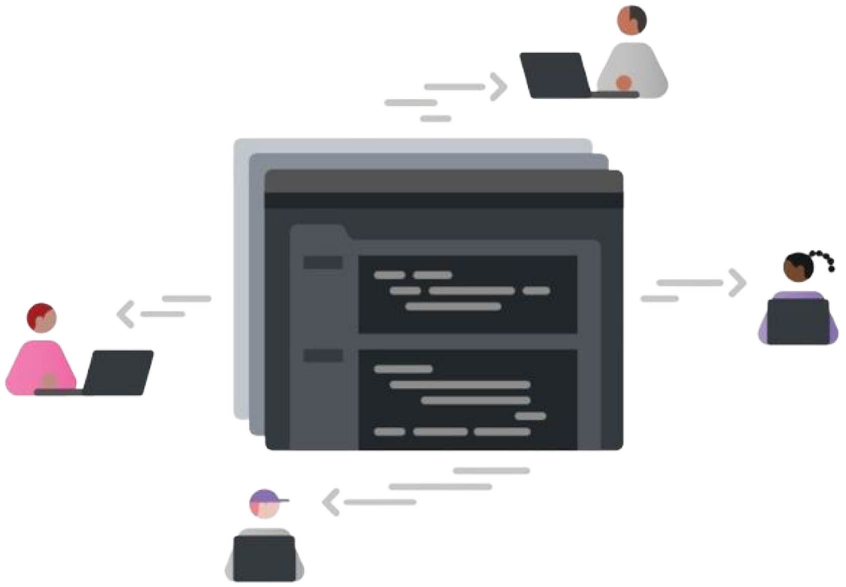
The kernel function value is computed as the probability of measuring the state $|0^{\otimes n}\rangle$ at the end of the circuit

$O(N^2)$ executions of the same PQC with different parameters values!

Qiskit ecosystem

The Qiskit ecosystem is a collection of tools created by researchers and developers who use Qiskit every day



https://www.ibm.com/quantum/ecosystem

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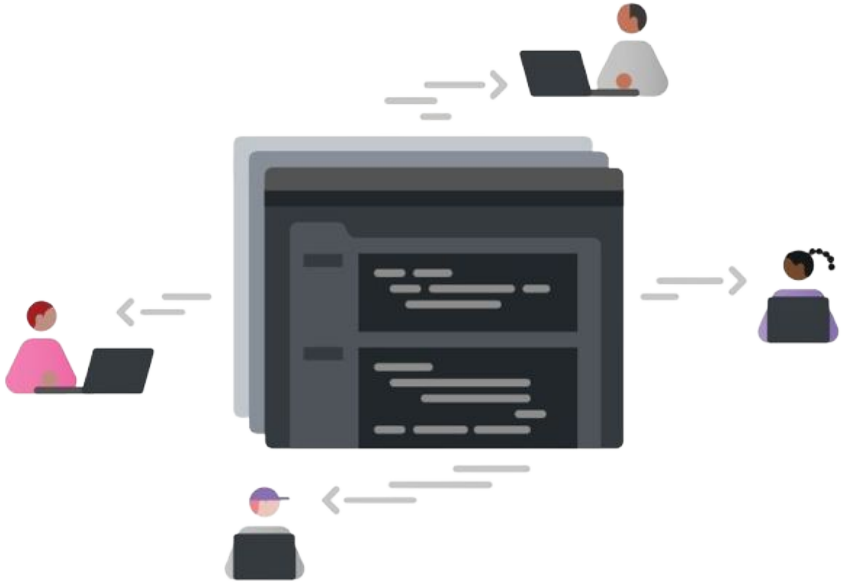

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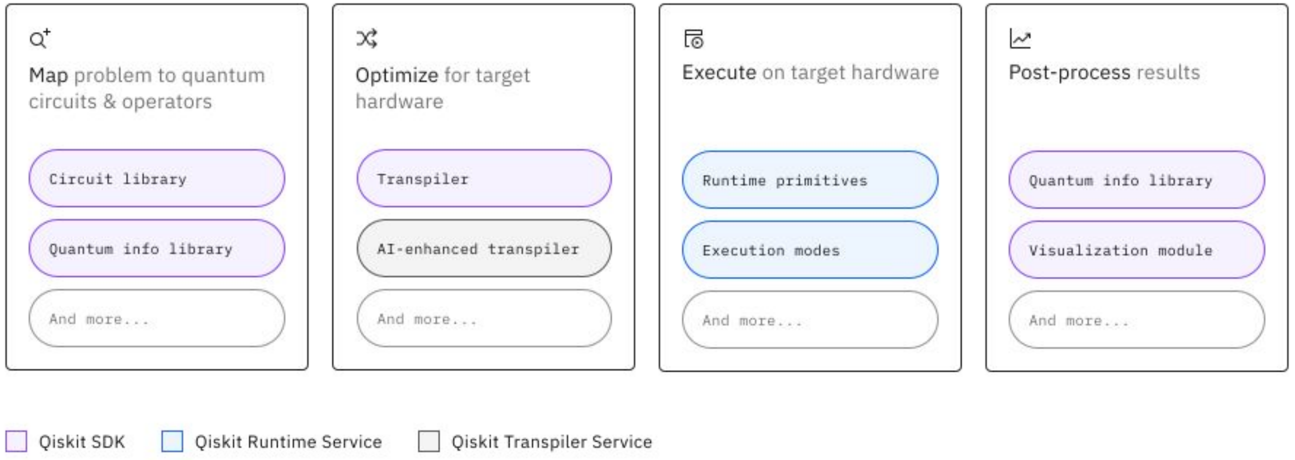

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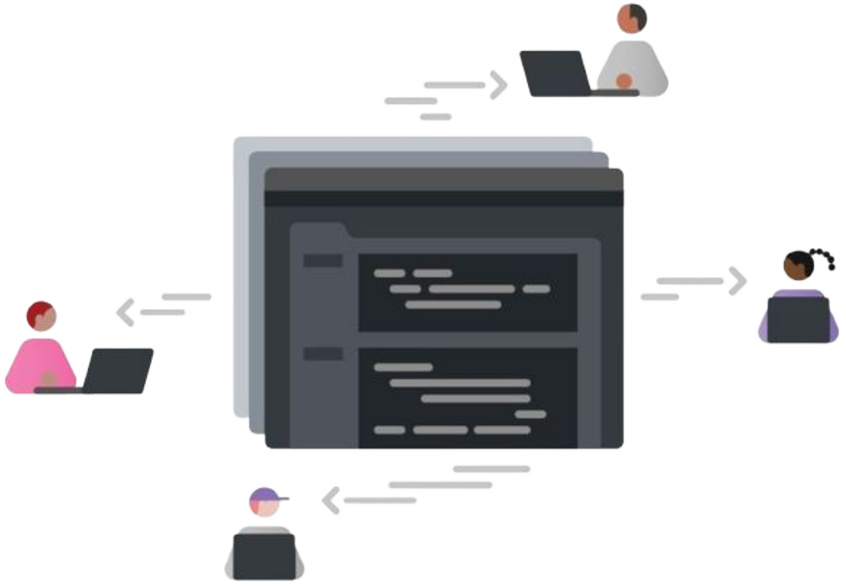

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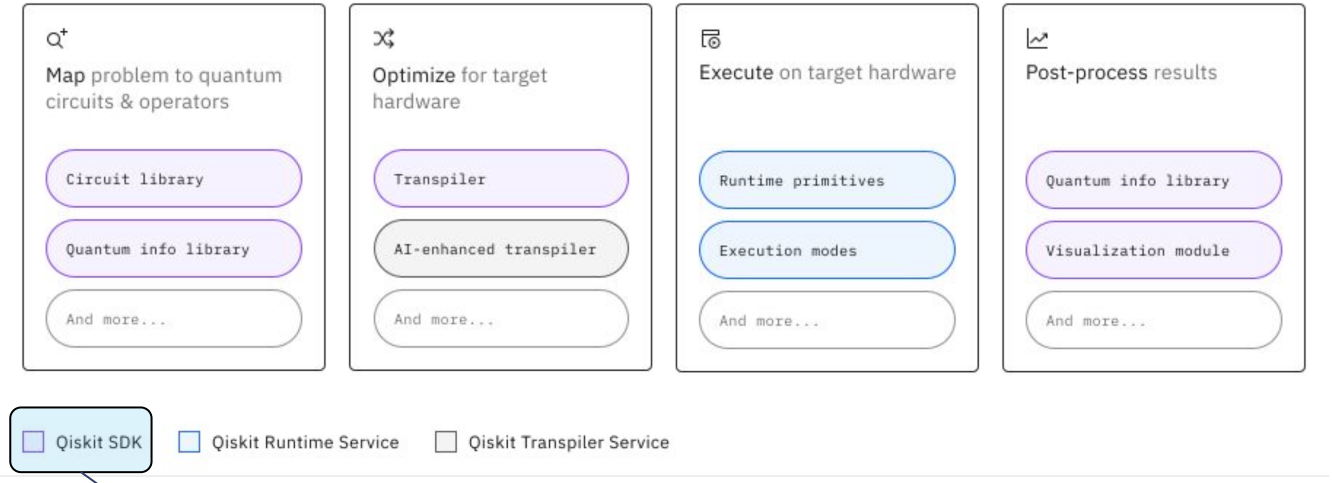
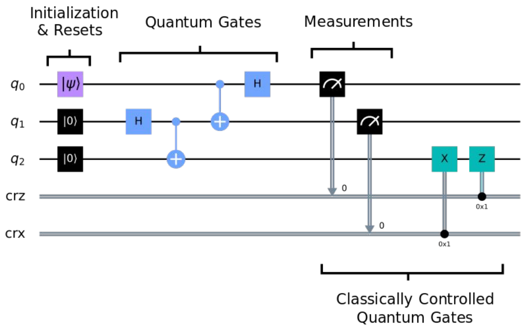
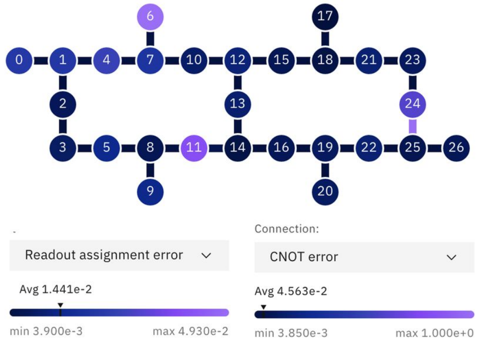


"The Qiskit SDK is an open-source SDK for working with quantum computers at the level of extended (static, dynamic, and scheduled) quantum circuits, operators, and primitives"



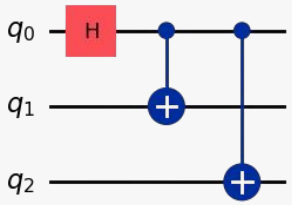


Quantum info library

```python
from qiskit import QuantumCircuit

qc = QuantumCircuit(3)
qc.h(0)
qc.cx(0, 1)
qc.cx(0, 2)

qc.draw('mpl')
```



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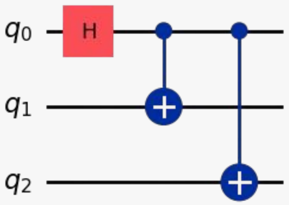


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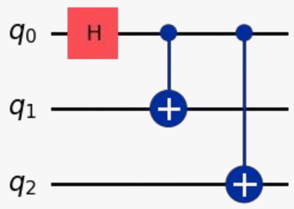


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from qiskit.quantum_info import Statevector

psi = Statevector(qc)
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$$
\begin{aligned}
|\psi\rangle &= \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \\
&= \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}
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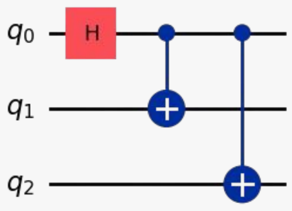


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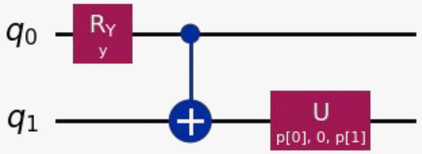
from qiskit import QuantumCircuit
from qiskit.circuit import Parameter, ParameterVector

y = Parameter('y')
p = ParameterVector('p', length=2)

pqc = QuantumCircuit(2)
pqc.ry(y, 0)
pqc.cx(0, 1)
pqc.u(p[0], 0, p[1], 1)

pqc.draw('mpl')
```



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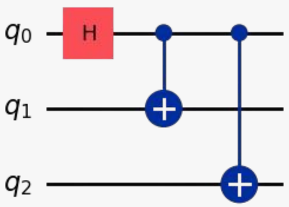


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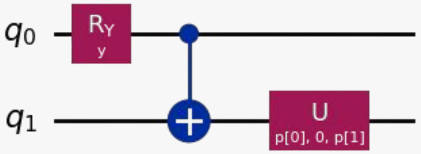


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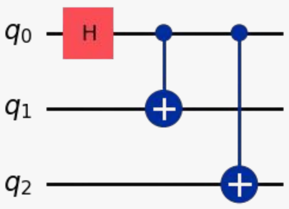


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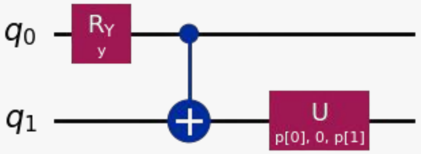


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```
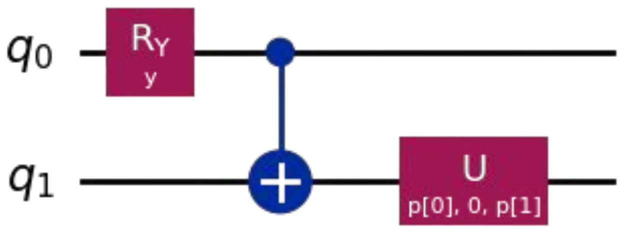

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
<ipython-input-7-032f9a8c4db1> in <cell line: 0>()
      1 from qiskit.quantum_info import Statevector
      2
----> 3 psi = Statevector(pqc)

7 frames
/usr/local/lib/python3.11/dist-packages/qiskit/circuit/parameterexpression.py in __float__(self)
    549         except (TypeError, RuntimeError) as exc:
    550             if self.parameters:
--> 551                 raise TypeError(
    552                     f"ParameterExpression with unbound parameters ({self.parameters}) "
    553                     "cannot be cast to a float."

TypeError: ParameterExpression with unbound parameters (dict_keys([Parameter(y)])) cannot be cast to a float.
```

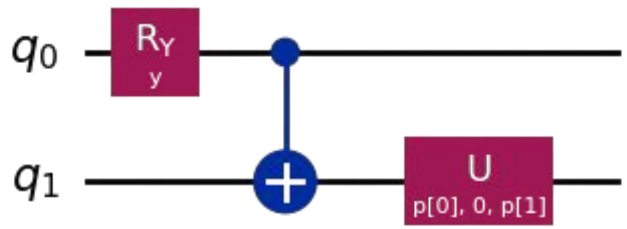
Qiskit-symb solution




```python
from qiskit_symb.quantum_info import Statevector

psi = Statevector(pqc)
psi.to_sympy()
```

$$\left[\cos\left(\frac{p[0]}{2}\right)\cos\left(\frac{y}{2}\right) \quad -e^{1.0ip[1]}\sin\left(\frac{p[0]}{2}\right)\sin\left(\frac{y}{2}\right) \quad \sin\left(\frac{p[0]}{2}\right)\cos\left(\frac{y}{2}\right) \quad e^{1.0ip[1]}\sin\left(\frac{y}{2}\right)\cos\left(\frac{p[0]}{2}\right)\right]$$

Qiskit-symb solution

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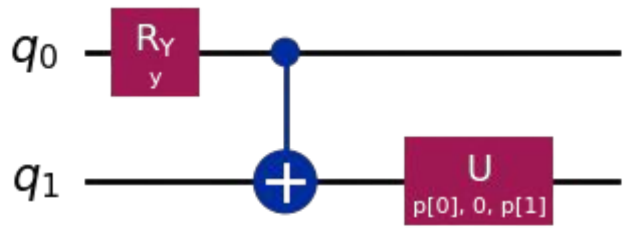
Consider a fixed PQC you need to execute many times with a different set of parameters values at each execution. You can use **qiskit-symb** to perform the (symbolic) linear algebra evaluation just once!

```python
from qiskit_symb.quantum_info import Statevector

psi = Statevector(pqc)
sim = psi.to_lambda()
```

Qiskit-symb solution



Qiskit symbolic

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But… There is much more!

Consider a fixed PQC you need to execute many times with a different set of parameters values at each execution. You can use **qiskit-symb** to perform the (symbolic) linear algebra evaluation just once!

```python
from qiskit_symb.quantum_info import Statevector

psi = Statevector(pqc)
sim = psi.to_lambda()
```

```python
import numpy as np

sim(np.pi/2, 0, 0)
```

```
array([ 0.70710678+0.j, -0.        +0.j,  0.70710678+0.j,  0.        +0.j])
```

Call Python lambda function to perform full-statevector simulation

Qiskit-symb performance

qiskit-symb time = **sympy eval** + **sympy lambdify** + **python run**

```
psi = Statevector(pqc)
```

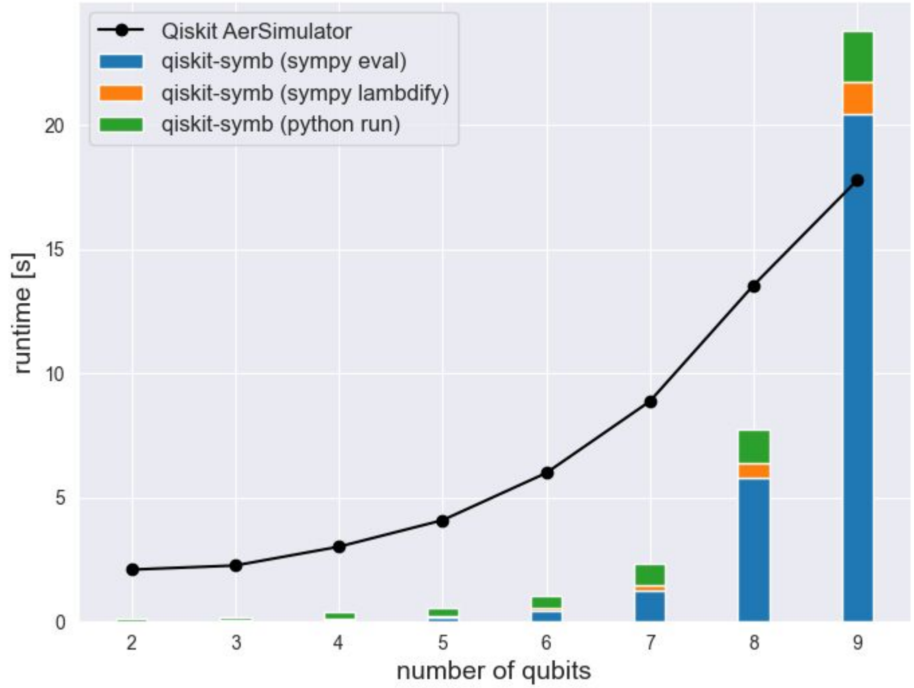
```
sim = psi.to_lambda()
```

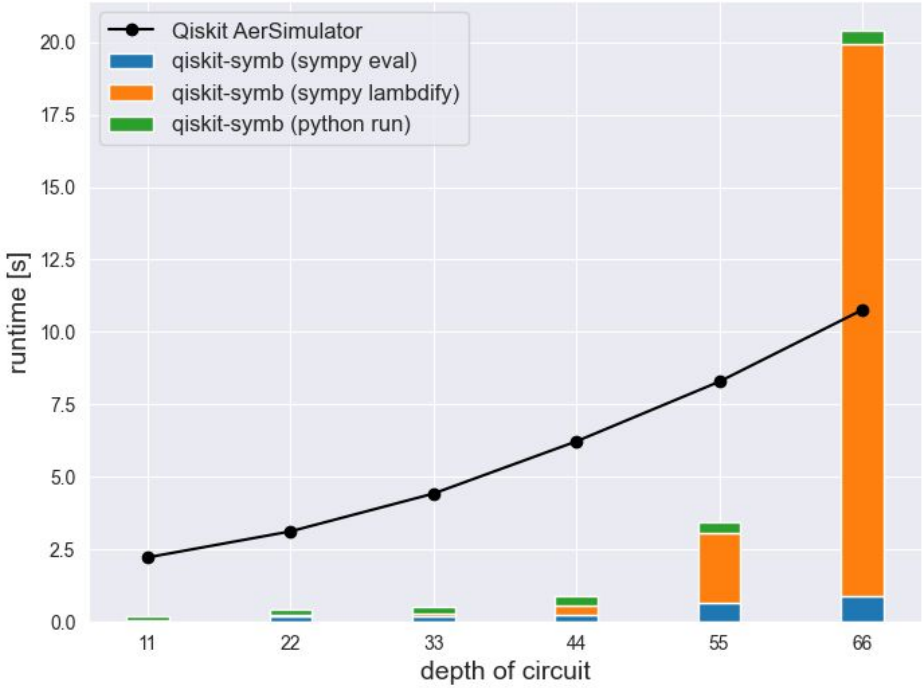
```
sim(*params)
```

Qiskit-symb performance

qiskit-symb time = **sympy eval** + **sympy lambdify** + **python run**

`psi = Statevector(pqc)` `sim = psi.to_lambda()` `sim(*params)`





The PQC used for the performance benchmark study is the `ZZFeatureMap` *provided by Qiskit. Each runtime refers to 10k executions of the same PQC with different random values assigned to the parameters at each execution.*

```
qiskit==1.2.4
qiskit-aer==0.15.1
qiskit-symb==0.4.0
```

Thank you!

Qiskit
symbolic

qiskit-symb Public

Symbolic evaluation of parameterized quantum circuits in Qiskit

GitHub

quantum-computing sympy symbolic-computation qiskit

Python 33 2 Apache License 2.0 2 issues need help Updated 2 weeks ago

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